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STATISTICAL, NONLINEAR, AND SOFT MATTER PHYSICS

Interaction of a Dielectric Macroparticle with a Point Charge in Plasma

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Abstract—The electrostatic interaction of a charged spherical dielectric macroparticle with a point charge in a plasma in the presence of an external uniform electric field is considered. The electrostatic force and the torque acting on the macroparticle have been determined, and the form of the interaction potential has been established for a nonuniform distribution of free charge on the macroparticle surface. A simple (for calculations) expression for the interaction potential that describes well the exact potential at all interparticle distances is proposed. The angular velocity of the spinning of dust particles caused by a nonuniform distribution of free charge over their surface has been estimated.

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1. INTRODUCTION

A dusty plasma is a convenient object for studying the properties of strongly coupled systems [1–3]. Many properties of such systems are determined by the interaction potential of charged macroparticles. In [4], we studied the interaction of a point charge with a charged conducting spherical body in a plasma by taking into account the Debye screening. We established that the interaction in the case where the spherical macroparticle charge remains constant during the approach differs greatly from the case where the surface potential remains constant. Here, this problem is generalized to the case of a dielectric macroparticle. We consider the general case of a nonuniform distribution of free charge on the macroparticle surface. For a constant macroparticle surface potential, the dielectric properties of the macroparticle play no role whatsoever, and this problem, basically, has already been solved in [4]. Therefore, in this paper, we consider only the case of a constant macroparticle charge. A nonuniform charge distribution can be maintained, for example, during the photoemission charging of dust particles by an external source of ultraviolet radiation or on dust particles levitating in the near-electrode layer of an RF discharge or in the cathode layer or strata in DC discharges, where electrostatic traps for dust particles are formed and there are strongly directed ion flows. The torque of the electrostatic forces acting on a macroparticle in a plasma was also determined.

2. THE FIELD OF A CHARGED MACROPARTICLE AND A POINT CHARGE IN PLASMA

Consider the interaction between a spherical macroparticle with radius a_1 and dielectric permittivity ε_1 and a point particle with charge q_2. Since under laboratory conditions dust particles usually levitate in regions with a sufficiently strong electric field that compensates the action of gravity, we will also include a constant electric field $\mathbf{E}_0$ in our analysis. Let us choose the origin of a Cartesian coordinate system at the macroparticle center, direct the z axis along the line connecting the particle centers, and direct the x axis in such a way that the vector $\mathbf{E}_0$ lies in the xz plane. Let us also introduce spherical coordinates, as shown in Fig. 1.

Suppose that there are no free space charges in region I ($r < a_1$). Therefore, the electric field potential in this region is defined by the Laplace equation [5]

$$\Delta\phi_{\mathrm{I}} = 0. \tag{1}$$

In region II ($r > a_1$), we will seek a self-consistent potential of the macroparticle and plasma based on a linearized Poisson–Boltzmann equation [6]:

$$\Delta\phi_1 - k_{\mathrm{D}}^2\phi_1 = 0, \tag{2}$$

where k_{D} is the reciprocal Debye length. Since the problem under consideration is linear, the total potential can be represented as

$$\phi = \begin{cases} \phi_{\mathrm{I}} & \text{in region I,} \\ \phi_{\mathrm{II}} \equiv \phi_0 + \phi_1 + \phi_2 & \text{in region II,} \end{cases} \tag{3}$$

where $\phi_0 = -\mathbf{E}_0 \cdot \mathbf{r} = -E_0 r\cos\beta$ is the constant electric field potential, β is the angle between the vectors $\mathbf{E}_0$

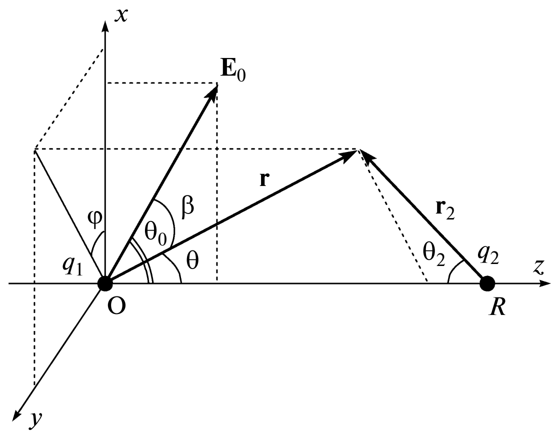

Fig. 1. Geometry of the point charge–macroparticle interaction problem: r, θ, φ are the coordinates of the point of observation in a spherical coordinate system with the origin at the macroparticle center; r_2, θ_2, φ are the coordinates of the point of observation in a spherical coordinate system with the origin at the location of the point charge; R is the interparticle distance; q_1, q_2 are the particle charges in elementary charges; $\mathbf{E}_0$ is the constant electric field vector lying in the xz plane; θ_0 is the angle specifying the direction of $\mathbf{E}_0$ in the spherical coordinate system with the origin at the macroparticle center; β is the angle between the vectors $\mathbf{E}_0$ and r.

and $\mathbf{r}$, and ϕ_2 is the self-consistent potential of a point particle in a plasma. The potential of a point particle in an equilibrium plasma is known to be the Debye one [6]:

$$\phi_2(r_2) = \frac{q_2}{\varepsilon r_2}\exp(-k_D r_2). \tag{4}$$

Note that $\varepsilon \approx 1$ in a plasma, but the dielectric permittivity of the medium ε in electrolytes or biological systems can differ from unity.

The boundary conditions for our problem are [5]

$$\phi_I\big|_{r=a_1} = (\phi_0 + \phi_1 + \phi_2)\big|_{r=a_1}, \tag{5}$$

$$\varepsilon_1 \frac{\partial \phi}{\partial r}\bigg|_{r=a_1-0} - \varepsilon \frac{\partial \phi}{\partial r}\bigg|_{r=a_1+0} = 4\pi\sigma(\theta, \varphi), \tag{6}$$

where σ is the macroparticle surface charge density.

The solutions of Eqs. (1) and (2) that are finite at zero and become zero at infinity in spherical coordinates are known to be [5, 7, 8]

$$\phi_I(r, \theta, \varphi) = \sum_{n=0}^{\infty}\sum_{m=0}^{n}(C_n^m\cos(m\varphi) + C_n^{-m}\sin(m\varphi))P_n^m(\cos\theta)r^n, \tag{7}$$

$$\phi_1(r, \theta, \varphi) = \sum_{n=0}^{\infty}\sum_{m=0}^{n}(A_n^m\cos(m\varphi) + A_n^{-m}\sin(m\varphi)) \times P_n^m(\cos\theta)\frac{K_{n+1/2}(k_D r)}{\sqrt{r}}, \tag{8}$$

where $K_{n+1/2}$ are modified Bessel functions of the second kind or half-integer-order Macdonald functions, and P_n^m are associated Legendre polynomials. Note that here the associated Legendre polynomials are defined without the Shortley–Condon factor (for example, $P_1^1 = \sin\theta$).

Let us expand the potential distribution for a point charge in a plasma (4) in terms of Legendre polynomials using Macdonald's formula [8], which at $r \le R$ is

$$\frac{\exp(-\tilde{r}_2)}{r_2} = \sum_{n=0}^{\infty}(2n+1)P_n(\cos\theta)\frac{I_{n+1/2}(\tilde{r})K_{n+1/2}(\tilde{R})}{\sqrt{rR}}, \tag{9}$$

where $I_{n+1/2}$ are modified Bessel functions of the first kind or half-integer-order Infeld functions,

$$r_2 = \sqrt{R^2 + r^2 - 2rR\cos\theta}, \quad \tilde{r} = k_D r, \quad \tilde{R} = k_D R.$$

The constant field potential is expanded as

$$\phi_0 = -E_0 r \times [\cos\theta_0 P_1^0(\cos\theta) + \sin\theta_0\cos\varphi P_1^1(\cos\theta)]. \tag{10}$$

Here, the angle θ_0 specifies the direction of the vector $\mathbf{E}_0$ in our spherical coordinate system (note that in our chosen coordinate system the azimuthal angle $\varphi_0 = 0$ for $\mathbf{E}_0$, see Fig. 1).

We will also expand the macroparticle surface charge density in terms of spherical harmonics:

$$\sigma(\theta, \varphi) = \sum_{n=0}^{\infty}\sum_{m=0}^{n}(\sigma_n^m\cos(m\varphi) + \sigma_n^{-m}\sin(m\varphi)) \times P_n^m(\cos\theta). \tag{11}$$

As a result, from the first boundary condition (5) we find

$$C_n^0 = A_n^0\frac{K_{n+1/2}(\tilde{a}_1)}{a_1^{n+1/2}} + q_2(2n+1) \times \frac{I_{n+1/2}(\tilde{a}_1)K_{n+1/2}(\tilde{R})}{\varepsilon a_1^{n+1/2}\sqrt{R}}, \quad n = 0, 2, 3, \ldots, \tag{12}$$

$$C_1^0 = A_1^0\frac{K_{3/2}(\tilde{a}_1)}{a_1^{3/2}} + 3q_2\frac{I_{3/2}(\tilde{a}_1)K_{3/2}(\tilde{R})}{\varepsilon a_1^{3/2}\sqrt{R}} - E_0\cos\theta_0, \tag{13}$$

$$C_1^1 = A_1^1\frac{K_{3/2}(\tilde{a}_1)}{a_1^{3/2}} - E_0\sin\theta_0, \tag{14}$$

$$C_n^{\pm m} = A_n^{\pm m}\frac{K_{n+1/2}(\tilde{a}_1)}{a_1^{n+1/2}}, \quad n = 1, 2, \ldots, \quad m = 1, 2, \ldots, n. \tag{15}$$

Here $\tilde{a}_1 = k_D a_1$. Using Eqs. (12)–(15), from the second boundary condition (6) we obtain

$$A_1^0 = \frac{3q_2}{\varepsilon\sqrt{R}}K_{3/2}(\tilde{R})M_{3/2}(\tilde{a}_1, \varepsilon, \varepsilon_1) + \frac{[4\pi\sigma_1^0 + (\varepsilon_1 - \varepsilon)E_0\cos\theta_0]a_1^{3/2}}{\varepsilon\tilde{a}_1 K_{5/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)K_{3/2}(\tilde{a}_1)}, \tag{16}$$

$$A_n^0 = \frac{(2n+1)q_2}{\varepsilon\sqrt{R}}K_{n+1/2}(\tilde{R})M_{n+1/2}(\tilde{a}_1, \varepsilon, \varepsilon_1) + \frac{4\pi\sigma_n^0 a_1^{3/2}}{\varepsilon\tilde{a}_1 K_{n+3/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)nK_{n+1/2}(\tilde{a}_1)}, \quad n = 0, 2, 3, \ldots, \tag{17}$$

$$A_1^1 = \frac{[4\pi\sigma_1^1 + (\varepsilon_1 - \varepsilon)E_0\sin\theta_0]a_1^{3/2}}{\varepsilon\tilde{a}_1 K_{5/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)K_{3/2}(\tilde{a}_1)}, \tag{18}$$

$$A_n^{\pm m} = \frac{4\pi\sigma_n^{\pm m} a_1^{3/2}}{\varepsilon\tilde{a}_1 K_{n+3/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)nK_{n+1/2}(\tilde{a}_1)}, \tag{19}$$

$$n = 1, 2, \ldots; \quad m = 1, 2, \ldots, n.$$

Here, for convenience, we introduced the function

$$M_{n+1/2}(\tilde{a}_1, \varepsilon, \varepsilon_1) = \frac{\varepsilon\tilde{a}_1 I_{n+3/2}(\tilde{a}_1) + (\varepsilon - \varepsilon_1)nI_{n+1/2}(\tilde{a}_1)}{\varepsilon\tilde{a}_1 K_{n+3/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)nK_{n+1/2}(\tilde{a}_1)}. \tag{20}$$

3. THE FORCE AND INTERACTION POTENTIAL OF A MACROPARTICLE AND A POINT CHARGE

The interaction potential of macroparticles in an isothermal plasma with a constant number of electrons and ions coincides with the free energy [9, 10]. To find the latter, let us first calculate the interaction force. The force acting on a point charged particle from a second finite-size particle can be calculated fairly easily (here, we do not consider the force acting on the macroparticle from a constant electric field, $q_1\mathbf{E}_0$, because we assume it to be balanced by the force of gravity):

$$\mathbf{F} = -q_2\nabla\phi\big|_{r=R,\,\theta=0} = -q_2\left(\frac{\partial\phi_1}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial\phi_1}{\partial\theta}\mathbf{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial\phi_1}{\partial\varphi}\mathbf{e}_\varphi\right)\Bigg|_{r=R,\,\theta=0}. \tag{21}$$

Hence, using Eq. (8), after simple calculations, for the z component of the force we find

$$F_z = eq_2\sum_{n=0}^{\infty}\frac{A_n^0}{R^{3/2}} \times [(n+1)K_{n+1/2}(\tilde{R}) + \tilde{R}K_{n-1/2}(\tilde{R})]. \tag{22}$$

The interaction potential does not depend on the path of integration. Therefore, integrating (22) over the interparticle distance, we will obtain an expression for the interaction potential:

$$U(R, \theta_0) = U_{\text{DLVO}} + U_{E0}\cos\theta_0 + \frac{4\pi a_1^2 q_2}{\sqrt{Ra_1}} \times \sum_{n=1}^{\infty}\frac{\sigma_n^0 K_{n+1/2}(\tilde{R})}{\varepsilon\tilde{a}_1 K_{n+3/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)nK_{n+1/2}(\tilde{a}_1)} + \frac{q_2^2}{2\varepsilon R}\sum_{n=0}^{\infty}(2n+1)K_{n+1/2}^2(\tilde{R})M_{n+1/2}(\tilde{a}_1, \varepsilon, \varepsilon_1). \tag{23}$$

Here, U_{DLVO} is the Derjaguin–Landau–Verwey–Overbeek (DLVO) potential (see, e.g., [4]):

$$U_{\text{DLVO}} = \frac{q_1 q_2}{\varepsilon R(1 + k_D a_1)}\exp\{-k_D(R - a_1)\} \equiv \frac{q_{1\text{eff}}q_2}{\varepsilon R}\exp(-\tilde{R}), \tag{24}$$

U_{E0} is the interaction potential of the surface charge induced by a constant external electric field with the point charge:

$$U_{E0} = \frac{a_1^2 q_2}{\sqrt{Ra_1}}\frac{(\varepsilon_1 - \varepsilon)E_0 K_{3/2}(\tilde{R})}{\varepsilon\tilde{a}_1 K_{5/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)K_{3/2}(\tilde{a}_1)}. \tag{25}$$

The third term on the right-hand side of Eq. (23) is the contribution from the interaction of the nonuniform surface charge with the point charge; the fourth term is the contribution from the interaction of the point charge with the surface charge induced by it.

For the interaction potential of a point particle and a uniformly charged macroparticle $\sigma_0^0 = q_1/4\pi a_1^2$ at $E_0 = 0$, we obtain

$$U(R) = U_{\text{DLVO}} + \frac{q_2^2}{2\varepsilon R}\left[\frac{I_{3/2}(\tilde{a}_1)}{K_{3/2}(\tilde{a}_1)}K_{1/2}^2(\tilde{R}) + \sum_{n=1}^{\infty}(2n+1)K_{n+1/2}^2(\tilde{R})M_{n+1/2}(\tilde{a}_1, \varepsilon, \varepsilon_1)\right]. \tag{26}$$

Note that in the case of $\varepsilon_1 \longrightarrow \infty$, i.e., if the macroparticle is a conductor, the formula

$$U(R) = U_{\text{DLVO}} + \frac{q_2^2}{2\varepsilon R}\left[\frac{I_{3/2}(\tilde{a}_1)}{K_{3/2}(\tilde{a}_1)}K_{1/2}^2(\tilde{R}) - \sum_{n=1}^{\infty}(2n+1)K_{n+1/2}^2(\tilde{R})\frac{I_{n+1/2}(\tilde{a}_1)}{K_{n+1/2}(\tilde{a}_1)}\right] \tag{27}$$

follows from (26). At $\varepsilon = 1$, it is transformed into the formula for the interaction potential of a conducting sphere and a point charge from [4].

If the interaction is considered in the absence of a plasma at $k_D = 0$, then, given the expansion of modified Bessel functions at small arguments (see [8]), from (21) and (26) we find

$$F \equiv F_z = \frac{q_1 q_2}{\varepsilon R^2} + \frac{q_2^2}{\varepsilon R^2} \times \sum_{n=1}^{\infty} \frac{n(n+1)(\varepsilon - \varepsilon_1)}{(n+1)\varepsilon + n\varepsilon_1}\left(\frac{a_1}{R}\right)^{2n+1}, \quad (28)$$

$$U(R) = \frac{q_1 q_2}{\varepsilon R} + \frac{q_2^2}{2\varepsilon R}\sum_{n=1}^{\infty} \frac{(\varepsilon - \varepsilon_1)n}{(n+1)\varepsilon + n\varepsilon_1}\left(\frac{a_1}{R}\right)^{2n+1}. \quad (29)$$

These expressions coincide with those for the force and interaction potential of a point charge with a dielectric sphere in a uniform dielectric [5].

4. NONUNIFORM DISTRIBUTION OF FREE CHARGE OVER THE MACROPARTICLE SURFACE

4.1. The Force and Interaction Potential

Let us now consider the case where there is a nonuniform charge distribution on the macroparticle surface. Since the charge on the macroparticle is produced by the flows of electrons and ions whose asymmetry is caused by the action of an electric field in the macroparticle levitation region, suppose that the charge distribution is axisymmetric along the constant electric field vector:

$$\sigma(\beta) = \sum_{n=0}^{\infty} \sigma_n P_n(\cos\beta).$$

Using the addition theorem for Legendre polynomials [11], for such a charge distribution we find

$$\sigma_n^0 = \sigma_n P_n(\cos\theta_0),$$
$$\sigma_n^m = 2\frac{(n-m)!}{(n+m)!}\sigma_n P_n^m(\cos\theta_0), \quad (30)$$
$$\sigma_n^{-m} = 0, \quad n = 1, 2, \ldots, \quad m = 1, 2, \ldots, n.$$

Below, we will dwell only on the case of a charge distribution with a predominance of the monopole and dipole moments, i.e., when $\sigma = \sigma_0 + \sigma_1\cos\beta$. In this case, from Eqs. (21) and (23) we find (note that $F_y \equiv 0$ for any axisymmetric charge distribution along the constant electric field vector, while an expression for the z component of the force can be easily derived from (22)):

$$F_x = -\frac{q_2 K_{3/2}(\tilde{R})\sin\theta_0 a_1^{3/2}}{R^{3/2}} \times \frac{(\varepsilon_1 - \varepsilon)E_0 + 4\pi\sigma_1}{\varepsilon\tilde{a}_1 K_{5/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)K_{3/2}(\tilde{a}_1)}, \quad (31)$$

$$U(R, \theta_0) = U_{\text{DLVO}} + (U_{E0} + U_{\sigma 1})\cos\theta_0 + \frac{q_2^2}{2\varepsilon R} \times \sum_{n=0}^{\infty}(2n+1)K_{n+1/2}^2(\tilde{R})M_{n+1/2}(\tilde{a}_1, \varepsilon, \varepsilon_1), \quad (32)$$

where

$$U_{\sigma 1} = \frac{q_2}{\sqrt{Ra_1}}\frac{4\pi a_1^2\sigma_1 K_{3/2}(\tilde{R})}{\varepsilon\tilde{a}_1 K_{5/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)K_{3/2}(\tilde{a}_1)}. \quad (33)$$

4.2. The Torque Acting on a Macroparticle

Under the assumption that the medium is in mechanical and thermal equilibrium, the following specific torque acts on a dielectric surface element [12]:

$$\mathbf{m} = \frac{\varepsilon}{4\pi}\left\{[\mathbf{r}\times\mathbf{E}](\mathbf{n}\cdot\mathbf{E}) - \frac{1}{2}E^2[\mathbf{r}\times\mathbf{n}]\right\} = \frac{\varepsilon a_1}{4\pi}E_r(E_\theta\mathbf{e}_\varphi - E_\varphi\mathbf{e}_\theta). \quad (34)$$

Using the recurrence relations for associated Legendre polynomials and their derivatives [11], for the projections of the torque on the Cartesian axes we find[1]

$$M_x = \int_0^{2\pi}\int_0^{\pi} m_x(\theta, \varphi)\sin\theta d\theta d\varphi = \frac{4}{3}\pi a_1^3\sigma_1^{-1}E_0 \times \cos\theta_0\frac{\varepsilon\tilde{a}_1 K_{5/2}(\tilde{a}_1)}{\varepsilon\tilde{a}_1 K_{5/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)K_{3/2}(\tilde{a}_1)} - \frac{q_2}{2} \times \sum_{n=1}^{\infty}\frac{4\pi n(n+1)\sigma_n^{-1}a_1^{3/2}}{\varepsilon\tilde{a}_1 K_{n+3/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)nK_{n+1/2}(\tilde{a}_1)} \times \frac{K_{n+1/2}(\tilde{R})}{\sqrt{R}}, \quad (35)$$

$$M_y = \int_0^{2\pi}\int_0^{\pi} m_y(\theta, \varphi)\sin\theta d\theta d\varphi = \frac{4}{3}\pi a_1^3 E_0\tilde{a}_1 \times K_{5/2}(\tilde{a}_1)\frac{\varepsilon(\sigma_1^0\sin\theta_0 - \sigma_1^1\cos\theta_0)}{\varepsilon\tilde{a}_1 K_{5/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)K_{3/2}(\tilde{a}_1)} + \frac{q_2}{2} \times \sum_{n=1}^{\infty} n(n+1)\frac{4\pi n\sigma_n^1 a_1^{3/2}}{\varepsilon\tilde{a}_1 K_{n+3/2}(\tilde{a}_1) + (\varepsilon_1 - \varepsilon)nK_{n+1/2}(\tilde{a}_1)} \quad (36)$$

[1] The calculation of the integrals is greatly simplified if we note that the vector in parentheses on the right-hand side of (34), to within a factor, is a result of the action of the orbital angular momentum operator on the electrostatic field potential, while the projection of this vector on the Cartesian coordinate axes is a result of the action of the corresponding projections of this operator on φ [13].

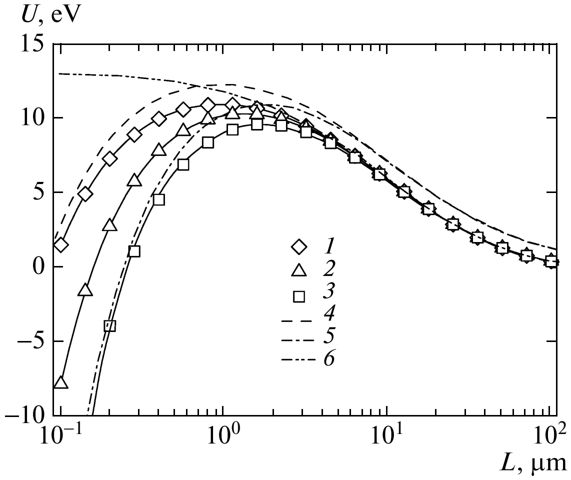

Fig. 2. Potential energy of the macroparticle–point charge interaction versus L for $q_1 = 10^3e$, $a_1 = 10$ μm, $q_2 = 10^2e$, and $k_D^{-1} = 50$ μm at various values of the dielectric permittivity: $\varepsilon_1 = 2$ (*1*), 4 (*2*), and 81 (*3*). The solid curves and symbols represent the calculations from (32) and (42), respectively; *4*—(29) at $\varepsilon_1 = 2$, *5*—(29) at $\varepsilon_1 = \infty$, *6*—DLVO potential (24).

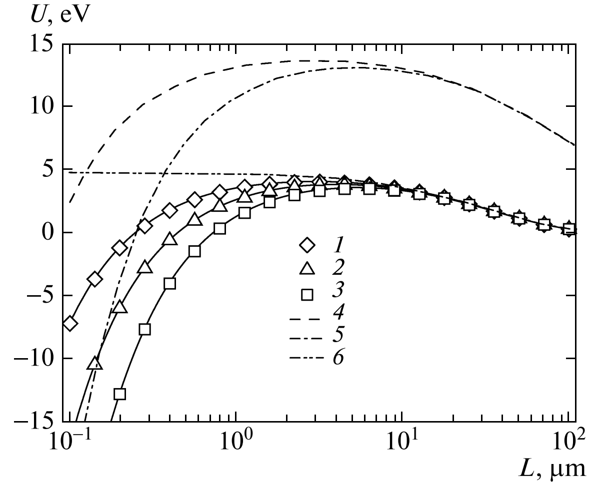

Fig. 3. Potential energy of the macroparticle–point charge interaction versus L for $q_1 = 10^4e$, $a_1 = 100$ μm, $q_2 = 10^2e$, and $k_D^{-1} = 50$ μm at various values of the dielectric permittivity: $\varepsilon_1 = 2$ (*1*), 4 (*2*), and 81 (*3*). The solid curves and symbols represent the calculations from (32) and (42), respectively; *4* – (29) at $\varepsilon_1 = 2$, *5*—(29) at $\varepsilon_1 = \infty$, *6*—DLVO potential (24).

$$\times \frac{K_{n+1/2}(\tilde{R})}{\sqrt{\tilde{R}}},$$

$$M_z = \int_0^{2\pi}\int_0^{\pi} m_z(\theta, \varphi)\sin\theta d\theta d\varphi \tag{37}$$
$$-\frac{(4/3)\pi a_1^3\varepsilon\sigma_1^{-1}E_0\sin\theta_0\tilde{a}_1K_{5/2}(\tilde{a}_1)}{\varepsilon\tilde{a}_1K_{5/2}(\tilde{a}_1) + (\varepsilon_1-\varepsilon)K_{3/2}(\tilde{a}_1)}.$$

For an axisymmetric (along the external field) distribution of free charge over the dust particle surface, from (35) and (37) we obtain

$$M_x = M_z = 0,$$

while the expression for the torque along the y axis will take the form

$$M_y = \frac{q_2}{2}\sum_{n=1}^{\infty}\frac{4\pi n(n+1)\sigma_n^1 a_1^{3/2}}{\varepsilon\tilde{a}_1K_{n+3/2}(\tilde{a}_1) + (\varepsilon_1-\varepsilon)nK_{n+1/2}(\tilde{a}_1)} \times \frac{K_{n+1/2}(\tilde{R})}{\sqrt{\tilde{R}}}. \tag{38}$$

Under the action of only the external field ($q_2 = 0$), from (35)–(37) we find

$$M_x = \frac{4}{3}\pi a_1^3\frac{\varepsilon\sigma_1^{-1}E_0\cos\theta_0\tilde{a}_1K_{5/2}(\tilde{a}_1)}{\varepsilon\tilde{a}_1K_{5/2}(\tilde{a}_1) + (\varepsilon_1-\varepsilon)K_{3/2}(\tilde{a}_1)}, \tag{39}$$

$$M_y = \frac{4}{3}\pi a_1^3\frac{\varepsilon E_0\tilde{a}_1K_{5/2}(\tilde{a}_1)(\sigma_1^0\sin\theta_0 - \sigma_1^1\cos\theta_0)}{\varepsilon\tilde{a}_1K_{5/2}(\tilde{a}_1) + (\varepsilon_1-\varepsilon)K_{3/2}(\tilde{a}_1)}, \tag{40}$$

$$M_z = -\frac{4}{3}\pi a_1^3\frac{\varepsilon\sigma_1^{-1}E_0\sin\theta_0\tilde{a}_1K_{5/2}(\tilde{a}_1)}{\varepsilon\tilde{a}_1K_{5/2}(\tilde{a}_1) + (\varepsilon_1-\varepsilon)K_{3/2}(\tilde{a}_1)}. \tag{41}$$

5. DISCUSSION

In Figs. 2 and 3, the potential energy of the interaction is plotted against $L = R - a_1$, the smallest distance between the macroparticle surface and the point charge, in various screening regimes (since only a plasma medium is considered below, we set $\varepsilon = 1$). We assumed the free charge to be distributed uniformly over the macroparticle surface. Our calculations showed that the interaction potential for a conducting macroparticle with $\varepsilon_1 = \infty$ is essentially identical to that for a particle with $\varepsilon_1 = 81$ at all interparticle distances.

It can be seen from Figs. 2 and 3 that repulsion is changed into attraction at small distances between the likely charged spherical macroparticle and the point charge, with this change at large dielectric permittivities occurring at large interparticle distances. In our calculations, we took into account the first 10^4 Legendre polynomial expansion terms (note that such a number of terms was required only at $L = 0.1$ μm, with the first discarded term having been smaller than the interaction potential itself by a factor of 10^{31}. The number of terms needed for the above accuracy to be achieved decreases rapidly with increasing interparticle distance). It was pointed out in [4] that, despite the larger (by a factor of 10) macroparticle charge in our calculations in a plasma with a stronger screening, the interaction energy at maximum turns out to be lower

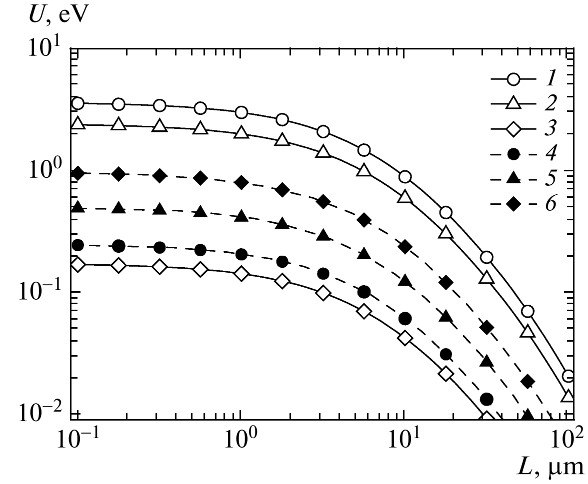

Fig. 4. Interaction potential of a nonuniform surface charge with a point charge (33) (curves *1–3*) and of the surface charge induced by a constant external electric field with a point charge (25) (curves *4–6*) versus L for $q_1 = -10^3e$, $a_1 = 10$ μm, $q_2 = 10^2e$, $k_D^{-1} = 100$ μm, $E_0 = 10$ V cm^{-1}, $\sigma_1 = \sigma_0 = q_1/4\pi a_1^2$ at various values of the dielectric permittivity: $\varepsilon_1 = 2$ (*1, 4*), 4 (*2, 5*), and 81 (*3, 6*).

than that in a less dense plasma. This is a consequence of the influence of both the macroparticle size (cf. curves *4* and *5* in Figs. 2 and 3) and the plasma screening ($U_{\rm DLVO}|_{L=0} = 4.8$ eV for a stronger screening and $U_{\rm DLVO}|_{L=0} = 12.9$ eV for a weaker one).

Figures 2 and 3 also present the curves calculated using the expression

$$U_a(R) = U_{\rm DLVO}(R) + \left(\frac{a_1}{R}\right)^2 \frac{\varepsilon_1 - \varepsilon}{\varepsilon_1 + 2\varepsilon} a_1 q_2 E_0 \cos\theta_0$$
$$+ \sum_{n=1}^{\infty} \left(\frac{a_1}{R}\right)^{n+1} \frac{q_2}{(n+1)\varepsilon + n\varepsilon_1} \tag{42}$$
$$\times \left[4\pi a_1 \sigma_n^0 + \frac{q_2}{2\varepsilon R}(\varepsilon - \varepsilon_1)n\right],$$

which was derived from (23) by retaining the screening effects only in the DLVO potential. We see that Eq. (42) describes well the potential energy of the interaction in a plasma at all distances.

Let us now turn to a nonuniform distribution of free charge over the macroparticle surface, namely to the case of an axisymmetric distribution along the external electric field. For simplicity, consider the case where only the monopole and dipole surface charge expansion terms are important. In this case, it can be seen from (31) that the component of the force along the x axis is nonzero. This force tends to rotate the point charge so that the direction of the straight line connecting the two particles coincides with the direction of the external electric field. At this position ($\theta_0 = 0$ or π), the interaction potential (32) as a function of the angle θ_0 has extrema: there will be a maximum at one value of θ_0 and a minimum at the other value. Precisely where the maximum and the minimum will depend both on the signs of q_2 and σ_1 and on E_0, σ_1, and ε_1.

Figure 4 presents the dependences $U_{E0}(L)$ and $U_{\sigma 1}(L)$ calculated from (25) and (33), respectively, at various values of the macroparticle dielectric permittivity. We see that the interaction of the dipole moment of the free charge with the point charge turns out to be more important at small ε_1, while the interaction of the surface charge induced by a constant external electric field with the point charge turns out to be more important as ε_1 increases. Note also that U_{σ_1} at small ε_1 is comparable to the interaction potential of a point charge with a uniformly charged dielectric and, therefore, should be taken into account when considering such phenomena as the coagulation of dust particles in a plasma. At the same time, U_{E0} becomes large at large ε_1.

In the levitation region of dust particles, the flow of positive plasma ions is directed along the external electric field. Therefore, one might expect the side of the macroparticle surface facing the ion flow to capture more ions than the opposite side and the flow of electrons on the macroparticle to be less sensitive to the external field and, in general, to be more isotropic because of the high electron temperature in a discharge. Consequently, the dipole moment of the free charge distribution over the macroparticle surface will be negative in the case of both negative and positive total charges. In this case, the terms U_{σ_1} and U_{E0} in (32) will be positive and to some extent will cancel each other out.

Figure 5 presents the dependences of the interaction potential for several values of the angle θ_0 calculated for a negatively charged macroparticle and a negative point charge. We see that the interaction potential for the parameters of the problem used in our calculations turns out to be minimal at $\theta_0 = \pi$ and maximal at $\theta_0 = 0$ for $\varepsilon_1 = 2$ and vice versa for $\varepsilon_1 = 81$.

Let us now discuss the spinning of dust particles that was observed in [14–17]. In the absence of a magnetic field, one might expect the surface charge distribution on the dust particles, on average in time, to be axisymmetric along the external electric field. For such a charge distribution, it follows from Eqs. (39)–(41) that

$$M_x = M_y = M_z = 0,$$

i.e., for an axisymmetric surface charge distribution, the torque acting on an isolated dust particle in the region of its levitation will be zero, in agreement with the conclusions reached in [17] about the absence of spinning of spherical dust particles in a discharge without a magnetic field. Note that for dust particles made of a conducting material, all torques will also become zero (in all of the above papers [14–17], the spinning of only dielectric dust particles was investigated).

When a magnetic field is applied, the axial symmetry of the problem will break, and a nonuniform charge distribution such that σ_1^{-1} and, possibly, $\sigma_1^0\sin\theta_0 - \sigma_1^1\cos\theta_0$ will become nonzero can appear in this case. Let us show this qualitatively. Let us direct the z axis along E_0 ($\theta_0 = 0$) and the x axis along the magnetic field. In [17], the magnetic field B did not exceed 300 G; in such fields, the electrons are magnetized, while the ions are essentially nonmagnetized. Therefore, the magnetic field under the conditions of the experiments [17] had virtually no effect on the current of ions in the layer (stratum), while in crossed electric and magnetic fields there is a drift current of electrons directed along the vector product $[\mathbf{E} \times \mathbf{B}]$, i.e., along the y axis. Note that this current at $B \approx 300$ G exceeds appreciably the drift current of electrons along the electric field. Consequently, we may write that the flow of ions on the macroparticle in the dipole approximation will be described by the dependence $J_i = J_{0i} - J_{1i}\cos\theta$, while the flow of electrons will be described with the same accuracy by the dependence $J_e = J_{0e} + J_{1e}\sin\varphi$.

Next, suppose that the electron charge density on the dust particle surface follows the angular dependence of the electron flow, while the ion charge density follows that of the ion flow. Then,

$$\sigma(\theta, \varphi) \approx \sigma_i + \sigma_e \sim \sigma_0 - \sigma_{1i}\cos\theta + \sigma_{1e}\sin\varphi.$$

For such a surface charge distribution,

$$\sigma_1^0 = -\sigma_{1i}, \quad \sigma_1^1 = 0, \quad \sigma_1^{-1} = \frac{3\pi}{8}\sigma_{1e}.$$

Consequently, as can be seen from (39)–(41), only the torque M_x will be nonzero. In the regime of weak screening, for the case under consideration we find

$$M_x \approx \frac{3\pi^2 a_1^3\sigma_{1e}E_0}{2(2+\varepsilon_1)}. \quad (43)$$

According to [18], the torque of the gas resistance for a rotating spherical particle is defined by the expression

$$M_{\rm fr} = \frac{2\pi}{3}\varrho_{\rm gas}v_{\rm th,\,gas}\omega_{\rm rot}a_1^4, \quad (44)$$

where $\varrho_{\rm gas}$ is the gas density, $v_{\rm th,\,gas}$ is the thermal velocity of the gas particles, and $\omega_{\rm rot}$ is the angular velocity of dust particle rotation. We will take into account the levitation condition for dust particles

$$q_1E_0 + m_dg = 0,$$

where m_d is the dust particle mass, and g is the acceleration of gravity. Let us also introduce a parameter defining the degree of anisotropy in the electron charge distribution over the dust particle surface:

$$\zeta_{1e} = \frac{4\pi a_1^2\sigma_{1e}}{q_1}.$$

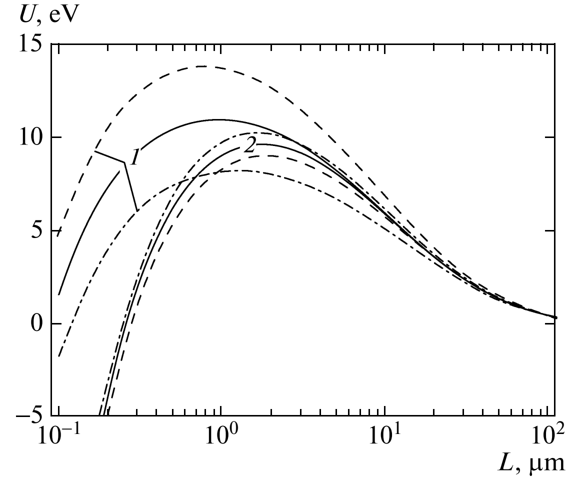

Fig. 5. Macroparticle–point charge interaction potential (32) versus L for $q_1 = -10^3e$, $a_1 = 10$ μm, $q_2 = -10^2e$, $k_D^{-1} = 100$ μm, $E_0 = 10$ V cm^{-1}, $\sigma_1 = \sigma_0 = q_1/4\pi a_1^2$ at various values of the dielectric permittivity: $\varepsilon_1 = 2$ (*1*) and 81 (*2*). The solid, dashed, and dash–dotted curves are for $\theta_0 = \pi/2$, 0, and π, respectively.

As a result, equating (44) and (43), for the angular velocity of dust particle rotation we find

$$\omega_{\rm rot} = -\frac{3\pi\zeta_{1e}}{4(2+\varepsilon_1)}\frac{\varrho_d}{\varrho_{\rm gas}}\frac{g}{v_{\rm th,\,gas}}, \quad (45)$$

where ϱ_d is the density of the dust particle material. Interestingly, the angular velocity of rotation in the approximation under consideration depends on the discharge parameters only via the parameter ζ_{1e} and does not depend explicitly on the dust particle size and charge.

Let us estimate the angular velocity of the spinning of dust particles for the conditions of the experiments [17], where the spinning of hollow spherical glass particles in neon at pressure $p = 0.15$ Torr was investigated. Suppose that

$$\zeta_{1e} = 10^{-2}, \quad T = 300\ \text{K}, \quad \varrho_d \approx 2\ \text{g cm}^{-3}, \quad \varepsilon_1 \approx 6.$$

We find from Eq. (45) that the angular velocity of rotation around the x axis (recall that we directed it along the magnetic field vector) is $\omega_{\rm rot} \approx 500$ rad s^{-1}. Such an angular velocity and the direction of the rotation axis are consistent with those obtained in [17], with such a large value being provided by only 1% electron charge anisotropy. A more rigorous consideration of the surface charge distribution in experiments is needed for a more accurate quantitative comparison.

6. CONCLUSIONS

Our analysis of the electrostatic interaction between a charged spherical dielectric macroparticle and a point charge in a plasma in the presence of an external uniform electric field showed that the interaction potential at all distances is well described by a superposition of the "long-range" DLVO potential and the "short-range" interaction potential of the dipole and higher moments of the distribution of the free and induced (by the external electric field and the point charge) surface charge with a point charge in a vacuum. We established that the point charge–macroparticle interaction potential has an extremum at the positions where the line connecting their centers is directed along the external electric field. In this case, the positions of the maximum and minimum depend on the relation between the external electric field and the dipole moment of the surface charge distribution. Our study of the torque of the electrostatic forces acting on a conducting macroparticle in a uniform external electric field showed that it is exactly equal to zero for a spherical particle and is nonzero for a dielectric macroparticle. Our estimates of the angular velocity of the spinning of dust particles caused by a nonuniform distribution of free charge over their surface showed that even a slight nonuniformity of the charge distribution over the surface could lead to significant angular velocities of the spinning of dielectric macroparticles.

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